The Competitive Industry in Short-Run Equilibrium: The Impact of Less than Perfectly Elastic Markets

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Abstract An individual competitive firm's original response to a price change may be reversed if that response triggers, at the industry level, other price adjustments in markets that are less than perfectly elastic. However, at the aggregate industry level such reversals will not occur provided abnormal reactions of these markets may be ruled out. The existence of less than perfectly elastic markets will merely weaken the industry's aggregate response. This note investigates the presumption of a systematic relationship between the size of price elasticities in an industry's less than perfectly elastic markets and the strength of its overall response to price changes in its perfectly elastic markets.

1 Introduction

If a competitive industry in short-run equilibrium, i.e. an industry with a fixed number of firms sells to, and buys from, both perfectly elastic as well as less than perfectly elastic markets, its original response to a price change in one of its perfectly elastic markets will typically not correctly describe its equilibrium response. For the original price change is likely to affect all the industry's input and output decisions, thereby triggering equilibrating price adjustments in its less than perfectly elastic markets. Its equilibrium response is, therefore, the sum of the original response and the reactions to these repercussions.

As was shown originally by Heiner (1982) and later generalized, these repercussions from less than perfectly elastic markets may reverse the original response of an individual firm, but not that of an entire industry, provided these less than perfectly elastic markets behave *normally* in a specific sense. Moreover, Heiner showed that the industry's total response to a price change is strongest if all its markets are perfectly elastic, gets weaker if some of its markets are less than perfectly elastic, and still weaker if these markets are completely inelastic. These interesting and, in their resemblance to Le Châtelier phenomena, also plausible results seem to suggest that an industry's equilibrium response to a price change weakens systematically as the price elasticities in its less than perfectly markets decrease. It is the purpose of this note to investigate this presumption.

2 The model of the industry

Consider a firm j with input-output vector $z^j = (x^j, y^j)$ where x comprises all inputs and outputs in infinitely elastic supply or demand while y represents all the remaining inputs and outputs originating from, or sold to, the industry's less than perfectly elastic markets. To simplify notation we will follow the convention to measure outputs along the positive and inputs along the negative axis. Let the price vector $p = (\alpha, \beta)$ that an individual firm faces be partitioned conformably.

The firms are not necessarily identical. They are assumed to be price-takers, maximizing their profit functions $\pi^j = p' z^j$. Denoting the profit-maximizing decision by $z^j(p)$, it is well known that an individual firm's response to an isolated price change

$$z_p^j = \begin{pmatrix} x_\alpha^j & x_\beta^j \\ y_\alpha^j & y_\beta^j \end{pmatrix} \tag{1}$$

is positive semidefinite and symmetric[1]. Using capital letters to denote summation over all firms, that is $Z = \Sigma z^j = (\Sigma x^j, \Sigma y^j) = (X, Y)$, (1) implies that the industry's aggregate response to an isolated price change

$$Z_p = \begin{pmatrix} X_\alpha & X_\beta \\ Y_\alpha & Y_\beta \end{pmatrix} \tag{2}$$

is likewise positive semidefinite and symmetric which, in turn, implies in particular

$$Y_\beta = Y_\beta' \tag{3}$$

and

$$X_\beta = Y_\alpha'. \tag{4}$$

While the X-markets are perfectly elastic and so always clear, the Y-markets are assumed to be less than perfectly elastic and, hence, clear only if the prices β are right. For these markets to clear, $Y(\alpha, \beta) = R(\beta)$ must hold, where $R(\beta)$ denotes the aggregate demand for the industry's outputs and the supply of its inputs in these less than perfectly elastic markets. Assuming that, for a given α, the corresponding market clearing prices $\beta = b(\alpha)$ are unique[2], we must have

$$Y(\alpha, b(\alpha)) = R(b(\alpha)). \tag{5}$$

In what follows, demand for the industry's outputs and supply of its inputs in the Y-markets are said to behave *normally* if R_β is negative semidefinite[3].

[1] Compare e.g. Silberberg (1978), pp. 284f.

[2] This implies that the matrix of derivatives, $Y_\beta - R_\beta$, is regular.

[3] Though it would be rather convenient, symmetry is not assumed because it cannot be justified on economic grounds. Negative semidefiniteness, however, is a

3 Less elastic supply and demand

Consider now two market constellations characterized by $R(\beta)$ and $\bar{R}(\beta)$, respectively, which differ only in that, at a given β^0 where $R(\beta^0) = \bar{R}(\beta^0)$ holds, $\bar{R}(\beta^0)$ is less elastic than $R(\beta^0)$. More precisely, the supply and demand functions $\bar{R}(\beta)$ are said to be *less elastic* than $R(\beta)$ at this point if the difference

$$R_\beta(\beta^0) - \bar{R}_\beta(\beta^0) = D \tag{6}$$

is negative semidefinite[4].

With these definitions, we may now investigate the industry's total price responses

$$X_\alpha^T = X_\alpha + Y_\alpha' b_\alpha \tag{7}$$

and

$$\bar{X}_\alpha^T = X_\alpha + Y_\alpha' \bar{b}_\alpha, \tag{8}$$

where the symmetry property (4) was used to substitute X_β and $\bar{b}_\alpha$ denote the equilibrium price adjustments in the less than perfectly elastic markets when $\bar{R}$ applies. Now, differentiating (5) as well as the corresponding equilibrium condition for the less elastic $\bar{R}$-markets, $Y(\alpha, \bar{b}(\alpha)) = \bar{R}(\bar{b}(\alpha))$, we have, after minor rearrangement,

$$Y_\alpha = (R_\beta - Y_\beta) b_\alpha \tag{9}$$

and

$$Y_\alpha = (\bar{R}_\beta - Y_\beta) \bar{b}_\alpha, \tag{10}$$

which imply that both $Y_\alpha' b_\alpha = b_\alpha'(R_\beta' - Y_\beta) b_\alpha$ in (7) and $Y_\alpha' \bar{b}_\alpha$ in (8) are negative definite[5]. Thus, the repercussions from the industry's less than perfectly elastic markets weaken its immediate response ($= X_\alpha$) but, as may be shown, they are not strong enough to reverse the sign, i.e. both X_α^T and $\bar{X}_\alpha^T$ remain positive semidefinite.

natural assumption. It guarantees that an isolated increase in one of the industry's supplies (or a decline in one of its demands) can never lead to an overall increase in the respective equilibrium price.

4 Again, symmetry is not assumed. The purpose of this definition is to ensure that, at the point β^0, the own-price responses $\partial R^i / \partial \beta_i$ are smaller than $\partial \bar{R}^i / \partial \beta_i$. Thus, at this point, the demand and supply functions $\bar{R}^i$ run more steeply than the corresponding functions R^i.

5 While R_β' and $\bar{R}_\beta'$ are negative semidefinite by the normality assumption and Y_β is positive semidefinite by the standard second order conditions, the differences $R_\beta' - Y_\beta$ and $\bar{R}_\beta' - Y_\beta$ must be definite since we assumed uniqueness; see fn. 2.

Equating (9) and (10) and solving for $\bar{b}_\alpha$, we have

$$\bar{b}_\alpha = (\bar{R}_\beta - Y_\beta)^{-1}(R_\beta - Y_\beta)b_\alpha. \tag{11}$$

Thus, for the difference of (7) and (8) we may write

$$\begin{aligned} X^T_\alpha - \bar{X}^T_\alpha &= Y'_\alpha(b_\alpha - \bar{b}_\alpha) \\ &= b'_\alpha(R'_\beta - Y_\beta)(I - (\bar{R}_\beta - Y_\beta)^{-1}(R_\beta - Y_\beta))b_\alpha \\ &= b'_\alpha(R'_\beta - Y_\beta)(I - (\bar{R}_\beta - Y_\beta)^{-1}(\bar{R}_\beta - Y_\beta + D))b_\alpha \\ &= -b'_\alpha(R'_\beta - Y_\beta)(\bar{R}_\beta - Y_\beta)^{-1}Db_\alpha, \end{aligned} \tag{12}$$

where I denotes the identity matrix and use was made of (9), (11) and (6). The difference $X^T_\alpha - \bar{X}^T_\alpha$ is consequently a quadratic form in the product of two negative definite and one negative semidefinite matrices. However, while the inverse of a negative definite matrix is also negative definite, the product of two negative definite matrices need not be positive definite. In general, it remains, therefore, entirely unclear whether $X^T_\alpha - \bar{X}^T_\alpha$ is indeed positive semidefinite, as the presumption requires. As a matter of fact, examples may easily be found where the presumption is wrong[6].

The key reason for the presumption to fail is the lack of symmetry of the matrices R_β or $\bar{R}_\beta$. If either were symmetric, the presumption would hold. To see this, assume, for example, that $\bar{R}_\beta$ is symmetric and consider the quadratic form

$$\begin{aligned} &(b_\alpha - \bar{b}_\alpha)'(Y_\beta - \bar{R}_\beta)(b_\alpha - \bar{b}_\alpha) \\ &\quad = b'_\alpha(Y_\beta - \bar{R}_\beta)(b_\alpha - \bar{b}_\alpha) + \bar{b}'_\alpha(\bar{R}_\beta - Y_\beta)(b_\alpha - \bar{b}_\alpha) \\ &\quad = b'_\alpha(Y_\beta - \bar{R}_\beta)(b_\alpha - \bar{b}_\alpha) + X^T_\alpha - \bar{X}^T_\alpha, \end{aligned} \tag{13}$$

[6] Consider the following example which we owe to Wilhelm Forst, University of Ulm. Let

$$R_\beta - Y_\beta = \begin{pmatrix} -a & -1 \\ 1 & -a \end{pmatrix} \quad \text{and} \quad \bar{R}_\beta - Y_\beta = \begin{pmatrix} -b & -c \\ c & -b \end{pmatrix}$$

and the corresponding matrices

$$D = \begin{pmatrix} b-a & c-1 \\ 1-c & b-a \end{pmatrix} \quad \text{and} \quad (\bar{R}_\beta - Y_\beta)^{-1} = \frac{\bar{R}'_\beta - Y_\beta}{b^2 + c^2}.$$

Clearly, for $a > b > 0$, all of these matrices are negative definite. Setting $a = 2/3$, $b = 1/3$ and $c = 1$, matrix D is diagonal and the product of the three critical matrices in (12),

$$(R'_\beta - Y_\beta)(\bar{R}_\beta - Y_\beta)^{-1}D = \begin{pmatrix} 0,29 & 0,37 \\ -0,37 & 0,29 \end{pmatrix},$$

turns out to be positive definite. Thus, with these parameters, $X^T_\alpha - \bar{X}^T_\alpha$ is negative definite, contradicting the presumption.

which is clearly positive definite. To establish the presumption that $X_\alpha^T - \bar{X}_\alpha^T$ is positive semidefinite, it suffices to show that the first product in the last line, $b'_\alpha(Y_\beta - \bar{R}_\beta)(b_\alpha - \bar{b}_\alpha)$, is negative semidefinite. Now,

$$\begin{aligned} & b'_\alpha(Y_\beta - \bar{R}_\beta)(b_\alpha - \bar{b}_\alpha) \\ &= b'_\alpha(Y_\beta - \bar{R}_\beta)b_\alpha - b'_\alpha(Y_\beta - \bar{R}_\beta)\bar{b}_\alpha \\ &= b'_\alpha(Y_\beta - \bar{R}_\beta)b_\alpha - b'_\alpha(Y_\beta - R_\beta)b_\alpha \\ &= b'_\alpha(R_\beta - \bar{R}_\beta)b_\alpha \\ &= b'_\alpha D b_\alpha, \end{aligned} \tag{14}$$

and that is clearly negative semidefinite[7].

Thus, if the response in the less than perfectly elastic markets is symmetric either before or after the elasticities change, the industry's equilibrium response to price changes indeed strengthens as the elasticities in these less than perfectly elastic markets rise. However, as the example in fn. 6 also demonstrated, without symmetry of either R_β or $\bar{R}_\beta$ this property is not assured even in the purest case of an increase of elasticities in the less than perfectly markets, i.e. the case where only the own-price effects in the response matrices R_β and $\bar{R}_\beta$ change so that their difference, D, is diagonal.

4 Concluding remarks

Even without the symmetry assumption it remains, of course, true that the repercussions from less than perfectly elastic markets may weaken, but not reverse, the aggregate response of a competitive industry to a price change. Yet the presumption that this response gets weaker the less elastic these markets are - while holding for the transition from less than perfectly elastic markets to completely inelastic markets[8] - does not necessarily hold as well for the transition from less than perfectly elastic to still less elastic, but not yet completely inelastic, markets. Indeed, innocent examples may be constructed where the presumption turns out to be wrong. There appears then to exist no generally valid systematic relationship between the strength of an industry's response to price changes and the size of price elasticities in its less than perfectly elastic markets.

References

Heiner, Ron (1982) "The Theory of the Firm in Short-Run Industry Equilibrium," American Economic Review, **72**, 555-562.

[7] The proof for symmetric R_β works along the same lines. One merely has to take into account that symmetry of R_β implies symmetry of $b'_\alpha(Y_\beta - R_\beta)b_\alpha$, which in view of (9) and (10) implies, in turn, symmetry of $b'_\alpha(Y_\beta - \bar{R}_\beta)\bar{b}_\alpha$.

[8] Note that in the proof of this proposition the corresponding matrix $\bar{R}_\beta$ is the zero-matrix which is, trivially, symmetric.

Silberberg, Eugene (1978) The Structure of Economics, A Mathematical Analysis, New York etc.: McGraw-Hill.